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SOURCE-TYPE SOLUTIONS OF DEGENERATE DIFFUSION EQUATIONS WITH ABSORPTION

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ABSTRACT

The object of this paper is to study the existence of a solution of the Cauchy problem $u_t = \Delta u^m - u^p$, $u(x,0) = \delta(x)$, and when a solution exists, to study its behaviour as $t \to 0$.

1. Introduction

We consider the Cauchy problem

$$(1.1)\qquad \text{(I)}\begin{cases} u_t = \Delta(u^m) - u^p & \text{in } \mathbf{R}^n \times (0,\infty) \\ u(x,0) = \delta(x) & \text{in } \mathbf{R}^n \end{cases}$$

in which $m \geqq 1$, $p > 1$, $n \geqq 1$ and δ is the delta measure.

In a recent paper, Brezis and Friedman [4] showed that when $m = 1$, Problem I has a solution if and only if $p < 1 + (2/n)$.

The object of this paper is to study the existence and nonexistence of a solution of Problem I if $m > 1$ and, when a solution $u(x,t)$ exists, to study its behaviour as $t \downarrow 0$.

Suppose $m > 1$. We shall prove that

I. If $p > m + (2/n)$, then Problem I has no solution.

II. If $1 < p < m + (2/n)$, then Problem I has a solution.

III. Let $1 < p < m + (2/n)$, and let $u(x,t)$ be a solution of Problem I. Then

$$(1.2)\qquad t^{1/\beta}\{u(x,t) - E(x,t)\} \to 0 \qquad \text{as } t \to 0 \text{ uniformly,}$$

where

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(1.3) $$\beta = m - 1 + (2/n)$$

and E is the well-known Barenblatt–Pattle solution [2], [11] of the porous media equation

$$u_t = \Delta(u^m) \qquad \text{in } \mathbf{R}^n \times (0, \infty)$$

which satisfies the initial condition

$$u(x,0) = \delta(x) \qquad \text{in } \mathbf{R}^n.$$

The methods we use to prove I and II are different from those used by Brezis and Friedman. The first result will follow from applying the similarity transformation

(1.4) $$x' = x/k, \quad t' = t/k^{\beta n}, \quad u' = k^n u, \qquad k > 0$$

to a hypothetical solution of Problem I, and deducing a contradiction by letting $k \to 0$. This method has the advantage that it affords a very clear insight in the relation between the parameters m, p and n and their effect on the relative importance of the diffusion and absorption terms in the equation. It has the disadvantage that it does not yield the best result. In fact, by extending the method of Brezis and Friedman in a straightforward manner, one can show that if

$$p \geqq m + (2/n),$$

then Problem I has no solution. Thus, our method fails to yield the limiting case $p = m + (2/n)$.

The second result is proved by approximating the initial value $u_0(x) = \delta(x)$ by the sequence

$$\phi_l(x) = E(x, 1/l), \qquad l = 1, 2, \ldots$$

and showing that a subsequence converges to a solution of Problem I. Thus it is clear that the existence theorem we prove is restricted to the initial value u_0 being a Dirac mass, in contrast to the theorem proved by Brezis and Friedman for $m = 1$, which allows u_0 to be a measure.

Since we are particularly interested in the short time behaviour of solutions of Problem I, we shall not prove the uniqueness of solutions. For $m = 1$ it can be found in [4].

The third result, like the first, is proved by means of an analysis of the family of functions $u_k = u'$. It follows from the limiting behaviour of u_k as $k \to 0$.

For $m = 1$, we are able to reproduce the results of Brezis and Friedman [4] —

with u_0 restricted to a Dirac mass — by our method. The short time behaviour of solutions of Problem I is then found to be

$$t^{n/2}|u(x,t)-E(x,t)|\to 0 \qquad \text{as } t\to 0 \tag{1.5}$$

uniformly on sets $P_a\subset\mathbf{R}^n$ of the form

$$P_a(t)=\{x\in\mathbf{R}^n : |x|\leqq at^{1/2}\}, \qquad a\geqq 0, \quad t\geqq 0.$$

Here E is the fundamental solution of the heat equation. A more precise result than (1.5), obtained by a different method, can be found in the appendix of [5].

The large time behaviour of solutions of (1.1) for $m=1$ was discussed in relation to the asymptotic behaviour of the initial value $u_0(x)$ for large x by Gmira and Veron [8] and Kamin and Peletier [10]. For $m>1$ this question will be discussed in a forthcoming paper.

2. Preliminaries

Let $\mathbf{R}^+=(0,\infty)$, $S=\mathbf{R}^n\times\mathbf{R}^+$, and let for any $T>0$, $S_T=\mathbf{R}^n\times(0,T]$.

DEFINITION. A solution u of Problem I on $[0,T]$ is a nonnegative function defined in S_T such that

(i) $u\in C((0,T];L^1(\mathbf{R}^n))\cap L^\infty(\mathbf{R}^n\times[\tau,T])$ for every $\tau\in(0,T)$,

(ii) $\iint_{S_T}(\zeta_t u+\Delta\zeta u^m-\zeta u^p)dxdt=0$ for every $\zeta\in C_0^{2,1}(S_T)$,

(iii) $\lim_{t\downarrow 0}\int_{\mathbf{R}^n}u(x,t)\chi(x)dx=\chi(0)$ for every $\chi\in C_0^\infty(\mathbf{R}^n)$.

We shall compare the solution u of Problem I with the source type solution E of the porous media equation [2, 11, 14]:

$$E(x,t)=t^{-1/\beta}f(\eta), \qquad \eta=|x|/t^{1/\beta n}, \tag{2.1}$$

where

$$f(\eta)=c\,\{[\eta_0^2-\eta^2]_+\}^{1/(m-1)} \tag{2.2}$$

in which η_0 and c are positive constants, depending only on m and n, which have been chosen so that the total mass satisfies

$$\int_{\mathbf{R}^n}E(x,t)dx\equiv 1$$

and $[z]_+=\max\{0,z\}$.

LEMMA 1. *Let u be a solution of Problem* I. *Then $u\leqq E$ in S.*

PROOF. Let, for $\tau > 0$, the function w_τ be the solution of the problem

$$\begin{cases} w_t = \Delta(w^m) & \text{in } S, \\ w(x,0) = u(x,\tau) & \text{in } \mathbf{R}^n. \end{cases}$$

Since $u(\cdot,\tau) \in L^1(\mathbf{R}^n)$ for every $\tau > 0$, the existence and uniqueness of w_τ is ensured for every $\tau > 0$ [12].

Because u is a solution of Problem I,

$$w_\tau(x,0) \to \delta(x) \qquad \text{as } \tau \to 0 \quad \text{in } \mathcal{D}'.$$

Following the compactness arguments used by Pierre in [13], it can be shown that this implies that

$$w_\tau(x,t) \to E(x,t) \qquad \text{as } \tau \to 0 \quad \text{a.e. in } S.$$

By the Comparison Principle [3],

$$u(x, t+\tau) \leqq w_\tau(x,t) \qquad \text{in } S$$

for every $\tau > 0$, whence

$$u \leqq E \qquad \text{in } S.$$

Suppose u is a solution of Problem I on $[0, T]$. Then for $k > 0$ we define the function

$$u_k(x,t) = k^n u(kx, k^{\beta n} t),$$

where β is defined in (1.3). It follows by substitution that for every $k > 0$, u_k satisfies the equation

$$u_t = \Delta(u^m) - k^\mu u^p \tag{2.3}$$

where $\mu = n(m + (2/n) - p)$, and the initial condition

$$\lim_{t \downarrow 0} \int_{\mathbf{R}^n} u_k(x,t)\chi(x)dx = \lim_{t \downarrow 0} \int_{\mathbf{R}^n} u(y, k^{\beta n} t)\chi(y/k)dy = \chi(0) \tag{2.4}$$

for any $\chi \in C_0^\infty(\mathbf{R}^n)$. Thus, as before we find

LEMMA 2. *Let u_k be a solution of* (2.3) *and* (2.4) *on* $(0, T]$. *Then $u_k \leqq E$ in S_T.*

In the next lemma we use this bound to obtain an integral estimate for the functions u_k^q where $1 \leqq q < m + (2/n)$.

LEMMA 3. *Let u_k be a solution of* (2.3) *and* (2.4) *on* $(0, T]$, *and let*

$1 \leqq q < m + (2/n)$. *Then for any $R > 0$ there exists a constant $\mathscr{C}$, which only depends on m, n and R such that*

$$\int_0^T \int_{B_R} u_k^q \, dx dt \leqq \mathscr{C} T^{(m+(2/n)-q)/\beta}.$$

PROOF. By Lemma 2 and (2.1),

$$\begin{aligned}\int_0^T \int_{B_R} u_k^q \, dx dt &\leqq \int_0^T t^{-q/\beta} dt \int_{B_R} f^q(\eta) dx \\ &\leqq |\partial B_R| \int_0^T t^{-(q-1)/\beta} dt \int_0^\infty f^q(\eta) d\eta \\ &\leqq \mathscr{C} T^{(m+(2/n)-q)/\beta}\end{aligned}$$

because $q < m + (2/n)$ and hence $(q-1)/\beta < 1$.

3. Nonexistence

We shall assume that Problem I has a solution on some interval $(0, T_1]$, and show that if $p > m + (2/n)$, this leads to a contradiction. Note that u_k is then a solution of (2.3) on $R^n \times (0, T_k]$, when $T_k = T_1 / k^{\beta n}$.

LEMMA 4. *Let $T \in (0, \frac{1}{2}T_1)$ and $R > 0$. Then there exists a constant $\mathscr{C}$, which only depends on m, n, R and T_1 such that*

$$\int_0^T \int_{B_R} u_k^p \, dx dt \leqq \mathscr{C} k^{-\mu}. \tag{3.1}$$

PROOF. Let $\zeta \in C_0^{2,1}(S_{T_1})$. Then, by the definition of a solution of Problem I

$$k^\mu \int_0^{T_1} \int_{\mathbf{R}^n} \zeta u_k^p \, dx dt = \int_0^{T_1} \int_{\mathbf{R}^n} (\zeta_t u_k + \Delta \zeta u_k^m) dx dt. \tag{3.2}$$

Now choose $\zeta(x,t) = \eta(t)\chi(x)$, where $\eta \in C_0^\infty(0, T_1)$ satisfies $0 \leqq \eta \leqq 1$, $\eta(t) = 1$ for $0 < \tau \leqq t \leqq T$ and $\chi \in C_0^\infty(\mathbf{R}^n)$ satisfies $0 \leqq \chi \leqq 1$ in $\mathbf{R}^n$, $\chi = 1$ in B_R and $\chi = 0$ outside B_{2R}. Then

$$\int_0^{T_1} \int_{\mathbf{R}^n} \zeta_t u_k \, dx dt = \left(\int_0^\tau + \int_T^{T_1} \right) \eta'(t) dt \int_{\mathbf{R}^n} u_k(x,t) \chi(x) dx. \tag{3.3}$$

Because $u_k \in C((0, T]; L^1(\mathbf{R}^n))$, we obtain using (2.4):

$$\lim_{\tau \downarrow 0} \int_0^\tau \eta'(t) dt \int_{\mathbf{R}^n} u_k(x,t) \chi(x) dx = 1. \tag{3.4}$$

Hence, if we let $\tau \downarrow 0$ in (3.2) we obtain by means of (3.3) and (3.4)

$$k^{\mu}\int_0^T\int_{B_R} u_k^p dxdt \leqq 1+\int_T^{T_1}\int_{B_{2R}} |\zeta_t|\, u_k dxdt+\int_0^{T_1}\int_{B_{2R}} |\Delta\zeta|\, u_k^m dxdt.$$

By Lemma 3 the integrals on the right hand side are bounded, whence (3.1) follows.

From Hölder's inequality we deduce at once:

COROLLARY. *Let $T\in(0,\frac{1}{2}T_1)$ and $R>0$. Then there exists a constant $\mathscr{C}$, which only depends on m, n, R and T_1 such that*

$$\int_0^T\int_{B_R} u_k dxdt \leqq \mathscr{C}k^{-\mu/p}.$$

Thus, if $p>m+(2/n)$, $\mu<0$ and hence, for fixed T and R,

$$\int_0^T\int_{B_R} u_k dxdt \to 0 \qquad \text{as } k\to 0. \tag{3.5}$$

On the other hand, we have in view of the initial condition

LEMMA 5. *Let $T\in(0,T_1)$. Then for every $\varepsilon>0$, there exists a radius $R=R(\varepsilon)$ such that*

$$\liminf_{k\to 0}\int_0^T\int_{B_R} u_k dxdt \geqq (1-\varepsilon)T.$$

PROOF. Note that

$$\int_0^T\int_{B_R} u_k dxdt=\int_0^T\int_{\mathbf{R}^n} u_k dxdt-\int_0^T\int_{\mathbf{R}^n\setminus B_R} u_k dxdt. \tag{3.6}$$

By the initial condition we have

$$\int_0^T\int_{\mathbf{R}^n} u_k(x,t)dxdt=\int_0^T\int_{\mathbf{R}^n} u(y,k^{\beta n}t)dxdt\to T \qquad \text{as } k\to 0.$$

To estimate the last term in (3.6) we recall that by Lemma 2, $u_k\leqq E$. Therefore $u_k=0$ in $\mathbf{R}^n\setminus B_R$, and hence

$$\int_0^T\int_{\mathbf{R}^n\setminus B_R} u_k dxdt=0$$

for R sufficiently large.

Since Lemma 5 contradicts (3.5) we may draw the following conclusion.

THEOREM 1. *Let $m>1$ and $p>m+(2/n)$. Then Problem I has no solution.*

4. Existence

To prove the existence of a solution of Problem I for $p < m + (2/n)$, we approximate the initial measure $\delta(x)$ by a sequence of continuous functions $\{\phi_l\}$ and show that the corresponding sequence $\{u_l\}$ of solutions of equation (1.1) has a subsequence which converges to a solution of Problem I.

For the sequence $\{\phi_l\}$ we choose

$$\phi_l(x) = E(x, 1/l). \tag{4.1}$$

Then, by the properties of E,

$$\phi_l(x) \to \delta(x) \qquad \text{as } l \to \infty \quad \text{in } \mathcal{D}'.$$

Clearly, for every $l > 0$, ϕ_l is a continuous function with bounded support. Hence, by [3], the problem

$$(\mathrm{I}_l) \begin{cases} u_t = \Delta(u^m) - u^p & \text{in } S_T \\ u(x,0) = \phi_l(x) & \text{in } \mathbf{R}^n \end{cases}$$

has a unique solution u_l in the sense of section 2, except that condition (iii) is replaced by

$$u_l(x,t) \to \phi_l(x) \qquad \text{as } t \to 0 \quad \text{in } \mathcal{D}'.$$

By the Comparison Principle [3], and the special choice of the initial function ϕ_l, we have

$$u_l(x,t) \leqq E(x, t + 1/l) \qquad \text{for } x \in \mathbf{R}^n, \quad t \geqq 0. \tag{4.2}$$

Thus, by the properties (2.1) and (2.2) of E,

$$u_l(x,t) \leqq t^{-1/\beta} f(0) \qquad \text{for } x \in \mathbf{R}^n, \quad t \geqq 0 \tag{4.3}$$

for every $l > 0$, and

$$\operatorname{supp} u_l \subset \{(x,t) : |x| \leqq a(t+1)^{1/\beta n}, t \geqq 0\} \tag{4.4}$$

for every $l \geqq 1$.

In addition to the pointwise estimate (4.3), we need the following estimate. It gives a uniform (with respect to l) bound on the solutions u_l down to $t = 0$.

LEMMA 6. *Let $T > 0$, and suppose $1 \leqq q < m + (2/n)$. Then there exists a constant $\mathscr{C}$, which only depends on m and n, such that*

$$\int_0^T \int_{\mathbf{R}^n} u_l^q \, dx\, dt \leqq \mathscr{C} T^{(m+(2/n)-q)/\beta}. \tag{4.5}$$

The proof is similar to that of Lemma 3 and we omit it.

The uniform upper bound of the solutions u_l implies the following property:

LEMMA 7. *For every compact subset $K \subset S_T$, the sequence $\{u_l\}$ is equicontinuous in K.*

The proof of this lemma is almost the same as that of Proposition 1 of [6] ([7]), whence we omit it.

By Lemma 7, it is possible to select a subsequence, which we denote again by $\{u_l\}$, such that for every compact set $K \subset S_T$,

$$u_l \to u \qquad \text{in } C(K) \quad \text{as } l \to \infty. \tag{4.6}$$

The limit function u is defined and continuous on the whole of S_T, and has the properties

$$u(x,t) \leqq E(x,t) \qquad \text{in } S_T, \tag{4.7}$$

$$u \in C((0,T]; L^1(\mathbf{R}^n)) \cap L^\infty(\mathbf{R}^n \times [\tau, T]) \qquad \text{for every } \tau \in (0,T). \tag{4.8}$$

We are now in a position to prove the main result of this section.

THEOREM 2. *Suppose $1 < p < m + (2/n)$. Then Problem* I *has a solution on $[0,T]$ for every $T > 0$.*

PROOF. We shall show that the limit function u defined by (4.6) has the properties (i)–(iii) expected of a solution of Problem I.

Property (i) has been established in (4.8).

Property (ii) follows from the observation that, because u_l is a solution of Problem I_l

$$\int\int_{S_T} (\zeta_t u_l + \Delta\zeta u_l^m - \zeta u_l^p)\,dxdt = 0 \tag{4.9}$$

for any $\zeta \in C_0^\infty(S_T)$. If we let $l \to \infty$ through the subsequence we selected above, we find that u satisfies the required identity.

Thus it remains to prove the third property, i.e., that

$$\int_{\mathbf{R}^n} u(x,t)\chi(x)\,dx \to \chi(0) \qquad \text{as } t \to 0 \tag{4.10}$$

for every $\chi \in C_0^\infty(\mathbf{R}^n)$.

Let $0 < t_1 < t_2 < T$ and choose in (4.9) the test function $\zeta(x,t) = \eta(t)\chi(x)$, where $\eta \in C_0^\infty(0, T)$ has the properties $0 \leqq \eta \leqq 1$ and

$$\eta(t) = \begin{cases} 0, & 0 < t < t_1 - \tau \\ 1, & t_1 \leqq t \leqq t_2 \\ 0, & t_2 + \tau < t < T \end{cases}$$

in which $\tau > 0$ is chosen sufficiently small, and χ is any function in $C_0^\infty(\mathbf{R}^n)$. We substitute ζ into (4.9) and let $\tau \to 0$. In the limit we obtain

$$\left| \int_{\mathbf{R}^n} u_l(x, t_2)\chi(x)dx - \int_{\mathbf{R}^n} u_l(x, t_1)\chi(x)dx \right| \leqq A \int_{t_1}^{t_2} \int_{\mathbf{R}^n} u_l^p dxdt + B \int_{t_1}^{t_2} \int_{\mathbf{R}^n} u_l^m dxdt, \tag{4.11}$$

where $A = \| \chi \|_{L^\infty}$ and $B = \| \Delta\chi \|_{L^\infty}$. By Lemma 6, u_l^m and u_l^p are integrable over S_T. Hence, letting $t_1 \to 0$ in (4.11) we obtain, omitting the subscript 2 from t_2:

$$\left| \int_{\mathbf{R}^n} u_l(x, t)\chi(x)dx - \int_{\mathbf{R}^n} \phi_l(x)\chi(x)dx \right| \leqq \int_0^t \int_{\mathbf{R}^n} (Au_l^p + Bu_l^m)dxdt$$
$$\leqq \mathscr{C} t^{m+(2/n)-p},$$

where we have used Lemma 6 again. If we now let $l \to \infty$, we obtain

$$\left| \int_{\mathbf{R}^n} u(x, t)\chi(x)dx - \chi(0) \right| \leqq \mathscr{C} t^{m+(2/n)-p},$$

which completes the proof of Theorem 2.

Note that we have actually shown that the function $u : [0, T] \to L^1(\mathbf{R}^n)$ is Lipschitz continuous on compact subsets of $(0, T]$, and Hölder continuous, with exponent $m + (2/n) - p$, at $t = 0$.

5. Short time behaviour

In an earlier paper [10], we used the similarity transformation (1.4) to investigate the large time behaviour of solutions of Problem I when $m = 1$ and $p > 1 + (2/n)$. In this section we shall use a very similar method to study the short time behaviour of solutions of Problem I when $p < m + (2/n)$.

Thus, let u be a solution of Problem I on $[0, T]$ and let, for $k > 0$,

$$u_k(x, t) = k^n u(kx, k^{\beta n} t).$$

Unlike in [10], where we let $k \to \infty$, we shall now let $k \to 0$ to characterize the behaviour of $u(x, t)$ as $t \to 0$.

By Lemma 2, $u_k \leqq E$ in S_T for all $k > 0$. Hence the functions u_k are uniformly bounded in any strip $S_{\tau,T}$ with $0 < \tau < T$. Thus, as in section 4, we can use the result of [6] to conclude that there exists a subsequence $\{u_{k'}\}$, such that for every compact subset K of S_T

$$u_{k'} \to U \qquad \text{as } k' \to 0 \quad \text{in } C(K), \tag{5.1}$$

where U is some limit function in $C(S_T)$.

By Lemma 2, the functions u_k have compact support, so that for any $t \in (0, T]$,

$$u_{k'}(x,t) \to U(x,t) \qquad \text{as } k' \to 0 \quad \text{uniformly in } \mathbf{R}^n. \tag{5.2}$$

LEMMA 8. *$U = E$ a.e. in S_T.*

PROOF. We shall show that U is a solution of the problem

$$\text{(II)} \begin{cases} u_t = \Delta(u^m) & \text{in } S_T, \\ u(x,0) = \delta(x) & \text{in } \mathbf{R}^n. \end{cases}$$

Since E is also a solution of this problem, and there exists only one solution [13], we may deduce that $U = E$ in S_T.

Let $\zeta \in C_0^{2,1}(S_T)$. Then, for sufficiently small k, u_k is defined on S_T, and satisfies

$$\int\int_{S_T} (\zeta_t u_k + \Delta\zeta u_k^m) dxdt = k^\mu \int\int_{S_T} \zeta u_k^p \, dxdt.$$

By Lemma 3, the integral on the right hand side is uniformly bounded with respect to k. Hence, since $p < m + (2/n)$, and therefore $\mu > 0$, the right hand side vanishes as $k \to 0$. Thus, by (5.1) and the uniform boundedness of the functions u_k in strips $S_{\tau,T}$, $0 < \tau < T$, we find that

$$\int\int_{S_T} (\zeta_t U + \Delta\zeta U^m) dxdt = 0. \tag{5.3}$$

To prove that $U \in C([0,T]; L^1(\mathbf{R}^n) \cap L^\infty(S_{\tau,T}))$ for every $\tau \in (0,T)$, and that $U(\cdot, t) \to \delta$ as $t \to 0$ in $\mathcal{D}'$, one proceeds as in section 4, using Lemma 3 instead of Lemma 6. We omit the details.

COROLLARY. *The entire sequence $\{u_k\}$ converges to E as $k \to 0$.*

Now that the limit of the sequence $\{u_k\}$ as $k \to 0$ is known, we can give a description of the solution u of Problem I for short times.

It follows from (5.2) and the previous corollary that

$$u_k(x,1) = k^n u(kx, k^{\beta n}) \to E(x,1) \qquad \text{as } k \to 0$$

uniformly in $\mathbf{R}^n$. Hence, since $E(x,1) = k^n E(kx, k^{\beta n})$ for all $k > 0$

$$k^n\{u(kx, k^{\beta n}) - E(kx, k^{\beta n})\} \to 0 \qquad \text{as } k \to 0.$$

Setting $k^{\beta n} = t$, we obtain the following result.

THEOREM 3. *Suppose* $m > 1$ *and* $p < m + (2/n)$. *Let* u *be a solution of Problem* I. *Then*

$$t^{1/\beta}\{u(x,t) - E(x,t)\} \to 0 \qquad \text{as } t \to 0$$

uniformly in $\mathbf{R}^n$. *Here* $\beta = m - 1 + (2/n)$,

$$E(x,t) = t^{-1/\beta} f(\eta), \qquad \eta = |x|/t^{1/\beta n}$$

and

$$f(\eta) = c(n,m)\{[\eta_0^2 - \eta^2]_+\}^{1/(m-1)},$$

in which $c(n,m) = \{2m(n+2\gamma)\}^{-\gamma}$ $(\gamma = 1/(m-1))$ *and* η_0 *is chosen so that* E *has unit mass.*

REMARK. It follows at once from Theorem 3 that

$$t^{1/\beta} u(\cdot, t) - f = o(t) \qquad \text{as } t \to 0 \text{ uniformly.}$$

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